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A CONSTRAINED PERIODIC REVIEW MODEL FOR A PROBABILISTIC REPARABLE-ITEM INVENTORY SYSTEM

DAVID AINSWORTH HASTINGS

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A CONSTRAINED PERIODIC REVIEW MODEL FOR A PROBABILISTIC REPARABLE-ITEM INVENTORY SYSTEM

by

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Submitted in partial fulfillment of the
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ABSTRACT

A reparable-item inventory system has two sources of items to meet demands: from the procurement of new items, and from the repair of damaged or failed items. Further, the system contains two distinct inventories, one containing procured and repaired ready-for-issue items and the other containing those failed items awaiting repair. This thesis develops an approximate model of this general system assuming that the demand rate and return rate of failed items are probabilistic. A periodic review policy is assumed for procured items, and inductions of batches of failed items into repair are assumed to take place at regularly scheduled intervals of time. The model is structured as a single coordinated inventory system instead of two separate systems, one of procured items and one of repaired items. The optimal review period and "order up to" level for procured items are determined, along with the optimal time between repair batch inductions.

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TABLE OF SYMBOLS AND ABBREVIATIONS

A_P - Cost to make one procurement.

A_R - Set-up cost per repair cycle.

B - Annual budget.

$f(x, t)$ - Density function of the quantity of items, X, demanded in time t.

$g(y, t)$ - Density function of the quantity of items, Y, repaired and returned to RFI_R in time t.

h_1 - Holding cost for ready-for-issue items.

h_2 - Holding cost for non-ready-for-issue items.

J - Cost to make one review.

K - Cost of operating the inventory system.

NRFI - Non-ready-for-issue.

R - Procurement "order up to" quantity.

RFI_P - That portion of ready-for-issue stock that was procured.

RFI_R - That portion of ready-for-issue stock that came from repair.

S - Expected number of units backordered per unit time.

T - Review cycle time, expressed in years.

V - Number of units backordered.

$\bar{x}$ - Average annual demand rate.

$\bar{y}$ - Average annual return rate of repaired items to RFI_R.

η - Fixed cycle time for return of reparable items to ready-for-issue stock.

Π - Lagrange multiplier.

τ_P - Procurement leadtime, a given constant.

1. INTRODUCTION

There are many ways of categorizing items in an inventory system. One way is to put all items into one of two areas, either reparable or non-reparable. In the military, due to the nature of its business, there is a large number of reparable-type items that represents a considerable dollar inventory. Realistically, although an item has been classified as a reparable type of item, not all of a particular type that wear out or fail can be repaired. For example, the AN/PRC-6 radio is classified as reparable, but not all the AN/PRC-6 radios that fail or become inoperative can be economically restored to an operating condition. Because of this, items must be procured from time to time to replenish the overall inventory system. Understanding this, one realizes that demands can be satisfied with items that are either procured or that have been returned to a repair facility and repaired.

Many models have been formulated for the consumable-item inventory system. These models typically answer the questions of how much to procure and when to procure in order to minimize cost or shortages. However, when investigating a reparable-item inventory system, not only the questions of how much and when to procure must be answered, but also the questions of how much and when to repair must be answered. Further, the reparable-item inventory system should be viewed as a system, rather than separate inventories of procured and repaired ready-for-issue items.

The system discussed in this paper is reviewed periodically, and a procurement is made at the time of review. Demand, X, is a random variable with a density function $f(x, t)$ over the period t. The procurement leadtime, τ_P, is a constant. Repaired items are returned to the ready-for-issue stock at fixed time intervals of η, and the quantity returned during a time t is a random variable, Y, with the density function $g(y, t)$. Further, all backorders will be filled.

If the criterion of minimizing total cost per unit time to operate the system were used, a shortage cost would have to be postulated. This is extremely difficult to do when discussing military items. How many dollars does it cost the government or the people of the United States if a tank, jet fighter, or submarine is inoperable due to the lack of a spare part? Volumes of literature have been written attempting to answer this question, for example, Solomon, et al [5]. However, no one has yet provided a satisfactory method for assigning shortage costs for military equipment. For this reason, the chosen criteria is to minimize the number of units backordered, subject to an operating cost budget, and hence avoid postulating a shortage cost. This assumes that the problem of optimally segmenting an operating cost budget for an inventory control point into separate operating cost budgets for each type of item managed by the inventory control point is possible and has been accomplished.

This model is an approximate approach to the constrained periodic review system as described. The objective is to determine the optimal order up to level R for procurement, the optimal review period for procurement, T, and the optimal repair cycle time, η, such that the average number of backorders is minimized while maintaining the cost of operating the system less than or equal to the yearly budget.

2. MODEL FORMULATION

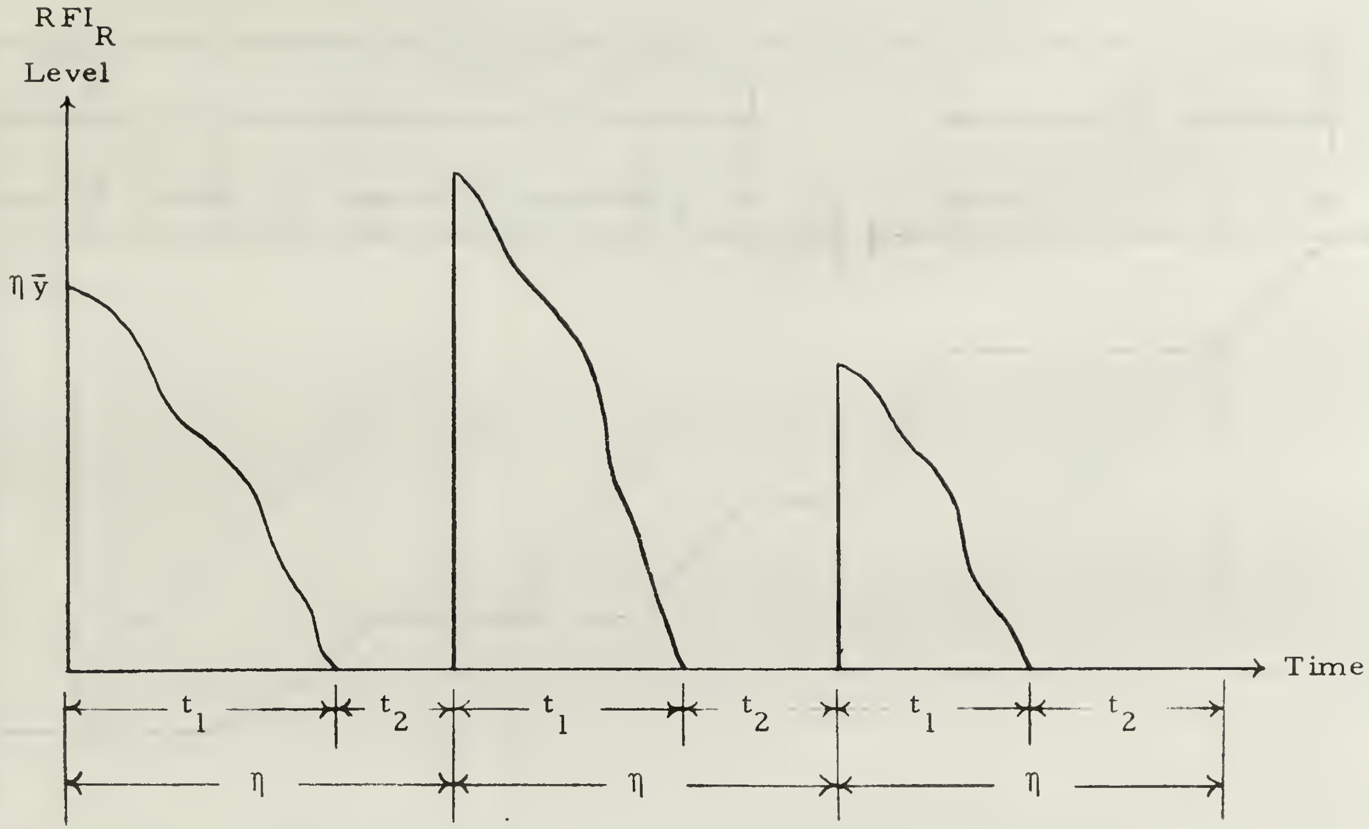
2.1 General

The on-hand inventory of ready-for-issue (RFI) stock may be viewed as two separate inventories, one consisting of all procured RFI items and the other consisting of all repaired RFI items. We may consider that all demands during η are filled by the RFI stock of repaired items (RFI_R) until it is depleted. At that time, demands are then placed on the procured RFI stock (RFI_P) until the end of the time period η. At the beginning of the next time period η, we receive a variable quantity of items for RFI_R, and the process starts over. Figures 1 through 3 portray this process and the accumulation of the non-ready-for-issue (NRFI) stock (the notation will be introduced below).

In order to minimize the expected number of units backordered, subject to a budget constraint on operating cost, the total variable operating cost per unit time must be determined. The relevant components of the variable cost per unit time needed to be determined are:

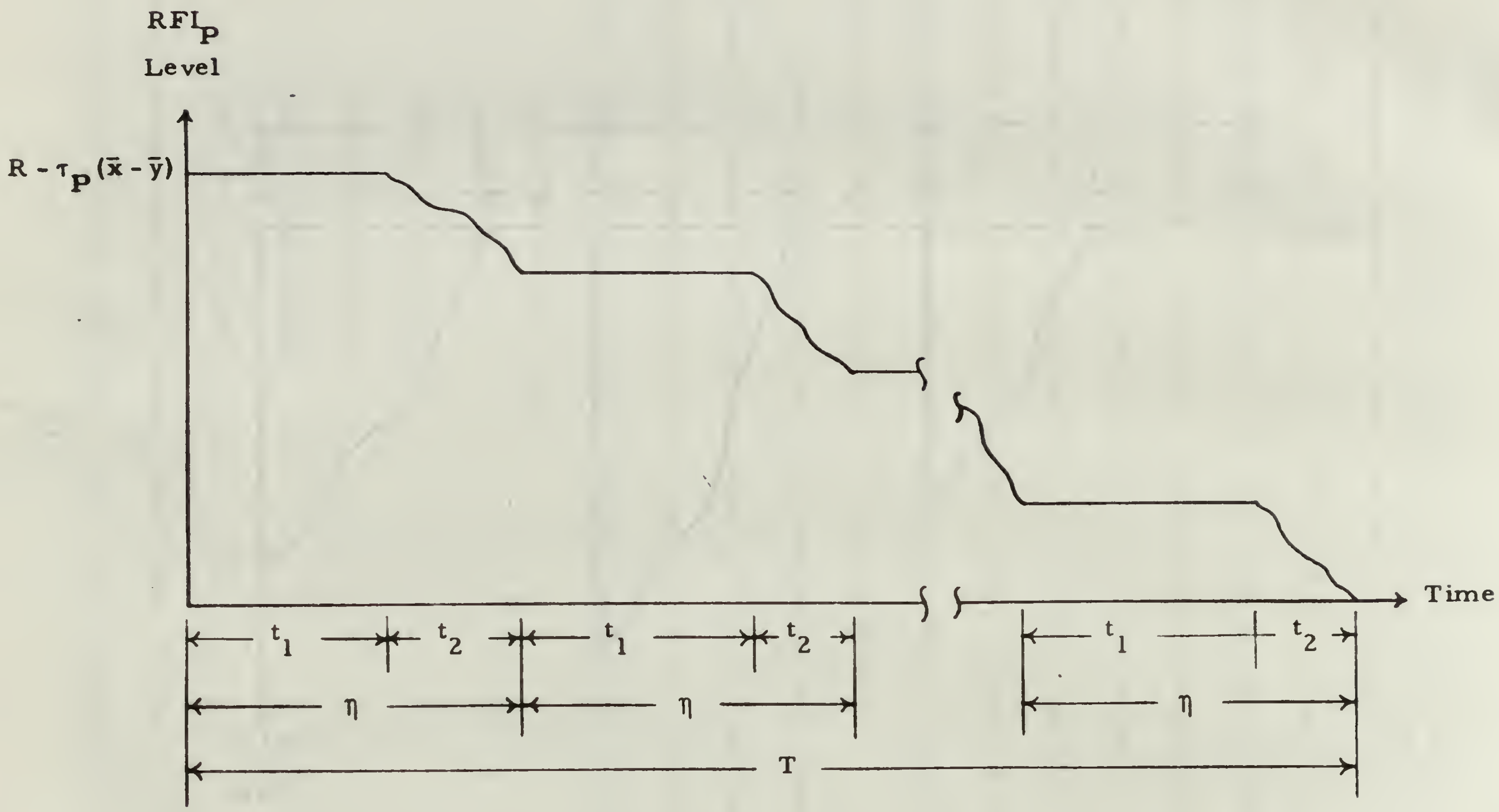
(a) procurement order cost;
(b) review cost;
(c) repair set-up cost;
(d) NRFI holding cost;
(e) RFI_R holding cost;
(f) RFI_P holding cost.

The following subsections develop these costs, the sum of which is the total variable cost per unit time. Then, the expression for determining



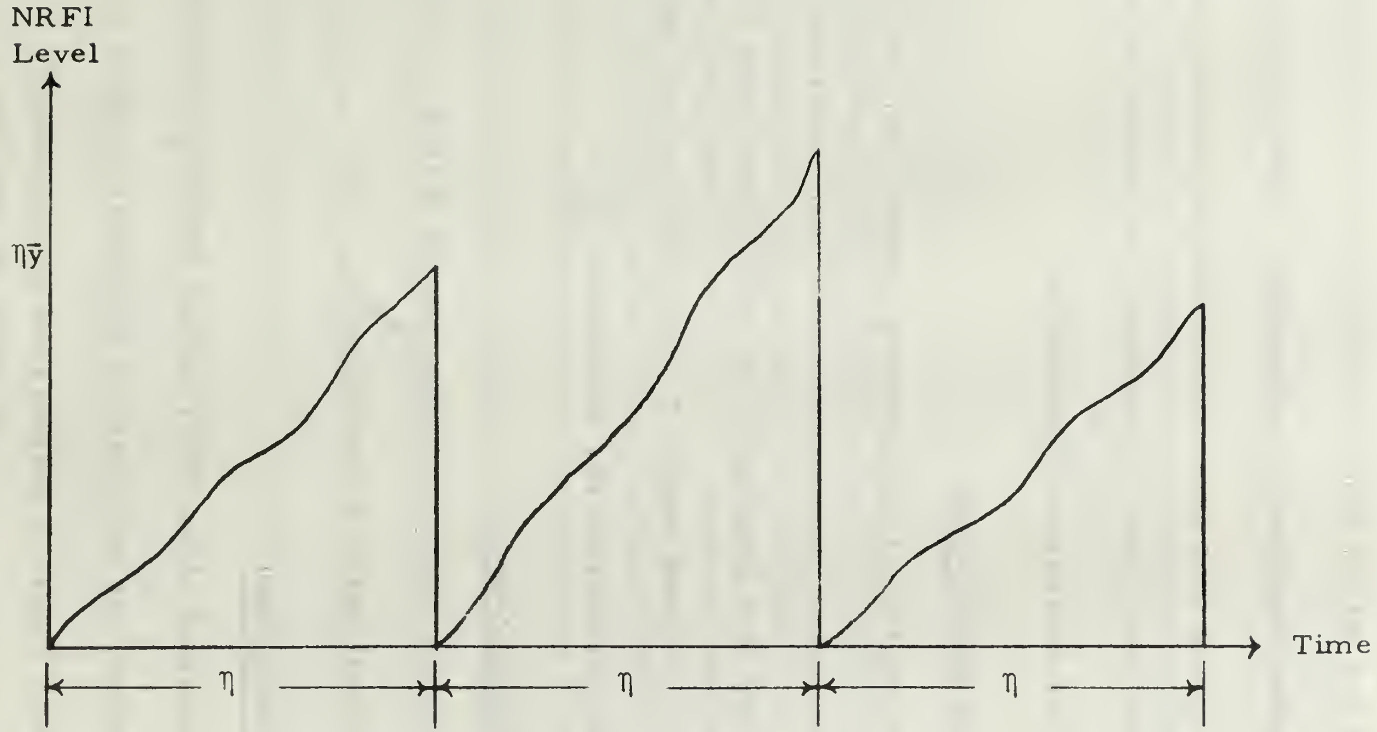
Repaired Ready-for-Issue Level Versus Time

FIGURE 1



Procured Ready-for-Issue Level Versus Time

FIGURE 2



Non-Ready-for-Issue Level Versus Time

FIGURE 3

the expected number of units backordered per unit time is developed. With this information, a solution to the problem may be determined. The formulation throughout is approximate, rather than exact. Inventory holding costs and set-up costs are determined by treating the expected values of random variables as parameters. The expected number of units short per unit time expression treats the random variables in the proper manner, but is only approximate for reasons which will be discussed when the expression is developed.

2.2 Order and Review Cost

In the description of the inventory system contained in the introduction, it was assumed that a procurement order was placed each time a review was made. The cost associated with placing one order is A_P. Since the review cycle is of fixed length, T, the order cost per unit time is $\frac{A_P}{T}$. Similarly, the review cost per unit time is $\frac{J}{T}$.

2.3 Repair Set-up Cost

The repair set-up cost per time period η is A_R. Therefore, the repair set-up cost per unit time is $\frac{A_R}{\eta}$.

2.4 NRFI Holding Cost

The dimensions of the NRFI holding cost, h_2, are dollars per unit year. Therefore, the unit years of stock held in NRFI inventory must be determined. Figure 3 portrays the NRFI inventory level over time. The average annual return rate of repaired items is $\bar{y}$. Since η is expressed in years, the expected height of each triangle is $\eta\bar{y}$ units.

The expected area of each triangle is then $\frac{\eta^2 \bar{y}}{2}$. The total number of unit years of NRFI held for the review period is $\frac{\eta^2 \bar{y}}{2}$ times the number of cycles of length η in one review period, $\frac{T}{\eta}$. Hence, the holding cost of NRFI per review period is

$$HC_T = h_2 \frac{\eta^2 \bar{y}}{2} \cdot \frac{T}{\eta} = \frac{h_2 \eta \bar{y} T}{2} .$$

Dividing by T yields the holding cost of NRFI per unit time:

$$HC = \frac{h_2 \eta \bar{y}}{2} . \qquad (1)$$

2.5 RFI_R Holding Cost

The dimensions of RFI_R holding cost, h_1, are dollars per unit year. Hence, to compute the RFI_R holding cost per unit time, the unit years of stock held must first be computed. Items are demanded at an average annual rate of $\bar{x}$ items. In the determination of NRFI holding cost, it was seen that the average number of items put into RFI_R stock per η was $\eta \bar{y}$. Because all items that are damaged or worn out cannot be repaired, we would normally expect the quantity demanded to exceed the quantity repaired. This information is portrayed in Figure 1.

The average amount of time, t_1, that $\eta \bar{y}$ will fill demands is $\frac{\eta \bar{y}}{\bar{x}}$. Thus, the holding cost per η is

$$HC_\eta = \frac{h_1 \eta \bar{y} t_1}{2} = \frac{h_1 \eta^2 \bar{y}^2}{2 \bar{x}} ;$$

and the holding cost per unit time is

$$HC = \frac{h_1 \eta \bar{y}^2}{2 \bar{x}} . \qquad (2)$$

2.6 RFI_P Holding Cost

Again, because of the dimensions of the holding cost, h_1, the unit years stocked must be computed. Rearranging the information in Figure 2 and including a buffer level, which is normally desirable when dealing with probabilistic demand, the RFI_P level may be portrayed as shown in Figure 4.

The inventory position, defined as the quantity of items on hand plus on order minus backorders, at the time a review is made is R. A time τ_P later, all units on order will have arrived and the inventory level will be R less the leadtime demand. The expected leadtime demand is $\tau_P(\bar{x} - \bar{y})$. Just prior to the arrival of the next procurement, one review period later, the inventory level will have decreased an amount equal to the period's demand. Hence, the inventory level just prior to the arrival of the next order is $R - \tau_P(\bar{x} - \bar{y}) - T(\bar{x} - \bar{y})$.

From the preceeding discussion of the RFI_R holding cost, it was noted that

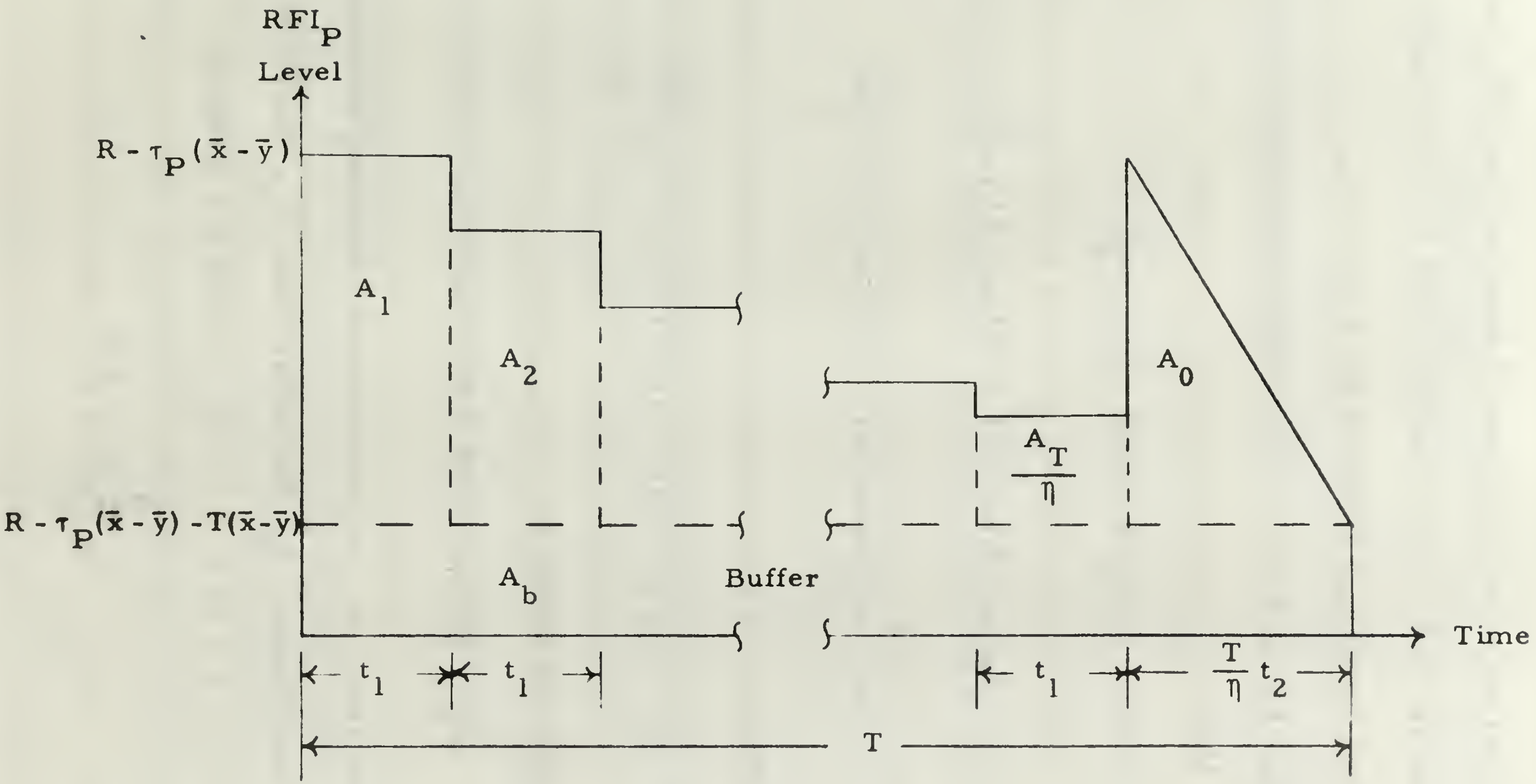
$$t_1 = \frac{\eta \bar{y}}{\bar{x}}$$

and

$$t_2 = \frac{\eta(\bar{x} - \bar{y})}{\bar{x}} .$$

Hence, the area of A_0 is

$$A_0 = \frac{T^2(\bar{x} - \bar{y})^2}{2\bar{x}} .$$



Rearrangement of Procured Ready-for-Issue Level Versus Time

FIGURE 4

During each period of η, the average number of items demanded upon RFI_P will equal the average number of demands that RFI_R could not satisfy, which is $\eta(\bar{x} - \bar{y})$. Therefore, the average height of each step in the step function in Figure 4 is $\eta(\bar{x} - \bar{y})$.

Each A_i, $i = 1, 2, \ldots, \frac{T}{\eta}$, will be the product of its height times t_1. Thus,

$$A_1 = [T(\bar{x} - \bar{y})] \frac{\eta\bar{y}}{\bar{x}} ,$$

$$A_2 = [T(\bar{x} - \bar{y}) - \eta(\bar{x} - \bar{y})] \frac{\eta\bar{y}}{\bar{x}} ,$$

$$A_3 = [T(\bar{x} - \bar{y}) - 2\eta(\bar{x} - \bar{y})] \frac{\eta\bar{y}}{\bar{x}} ,$$

and, in general,

$$A_i = [T(\bar{x} - \bar{y}) - (i - 1)\eta(\bar{x} - \bar{y})] \frac{\eta\bar{y}}{\bar{x}} .$$

The area of the buffer is

$$A_b = [R - \tau_P(\bar{x} - \bar{y}) - T(\bar{x} - \bar{y})] T .$$

The total area, A_T, is determined by summing the various areas: the area of the triangle, A_0; the buffer area, A_b; and the areas of the rectangles, A_i, $i = 1, 2, \ldots, \frac{T}{\eta}$. Using the relations

$$\sum_{j=1}^{N} P = PN$$

and

$$\sum_{j=1}^{N} j = \frac{N(N + 1)}{2} ,$$

the total area A_T may be determined to be

$$A_T = \frac{T(\bar{x} - \bar{y})(\eta\bar{y} - \bar{x}T)}{2\bar{x}} + T[R - \tau_P(\bar{x} - \bar{y})] \ .$$

One notes that the upper limit on the summation of the A_i's is $\frac{T}{\eta}$. This point may be questionable since there is no guarantee that $\frac{T}{\eta}$ will be an integer. However, if η is small compared to the review cycle, T, the error from considering $\frac{T}{\eta}$ as an integer is negligible.

The holding cost of RFI_P per review period is then $h_1 A_T$ and, dividing by the review period T, yields the holding cost per unit time.

$$HC = h_1 \left[R + \frac{(\bar{x} - \bar{y})(\eta\bar{y} - T\bar{x} - 2\bar{x}\tau_P)}{2\bar{x}} \right] . \tag{3}$$

Additionally, there may be some question as to whether or not the holding cost should be modified to account for the unit years of items backordered. Because this is an approximate model, we ignore this term. In light of the objective of this paper, this assumption is only valid when the budget constraint is not too restrictive.

2.7 Units Backordered

A procurement order placed at time t will arrive at $t + \tau_P$, and the next order will arrive at time $t + \tau_P + T$. At time t, after the order is placed, the inventory position is R. The next time the inventory position reaches R is at time $t + \tau_P + T$. Hence, shortages will occur if the demand on RFI_P during $\tau_P + T$ exceeds R. The number of units demanded on RFI_P is $X - Y$. Let the random variable Z, with density function $h(z, t)$, equal $X - Y$. Now, the number of units backordered, V, is

$$V = \begin{cases} 0 & \text{if } Z \leq R \\ Z - R & \text{if } Z > R \end{cases} .$$

Hence, the expected number of units backordered per unit time, denoted as S, is

$$S = \frac{1}{T} \int_R^{\infty} (Z - R)\, h(z, \tau_P + T)\, dz \quad . \tag{4}$$

This expression only accounts for the expected number of units backordered at the end of the time period $\tau_P + T$, and hence is only an approximation to the true expected number of units backordered during $\tau_P + T$. It does not consider the possibility that units may be backordered at the end of each time period η, and then filled by the input of repaired items into RFI_R. This possibility has been neglected since it is assumed that if this occurs, the number of items backordered and the length of time they are backordered are negligible. The duration of such shortages is only a fraction of η, which is assumed to be quite small. It should be noted that if the budget constraint is very restrictive, this assumption becomes unrealistic.

3. SOLUTION

As stated previously, the total variable operating cost per unit time of the system must be determined in order to minimize S, subject to a budget constraint. The variable cost per unit time, K, to operate the system is the sum of the various costs previously derived.

$$K = \frac{A_P + J}{T} + \frac{A_R}{\eta} + \frac{\eta \bar{y}(h_1 + h_2)}{2} + h_1\left[R - \frac{(\bar{x} - \bar{y})(T\bar{x} + 2\bar{x}\tau_P)}{2\bar{x}}\right] . \quad (5)$$

Because demand is probabilistic and the most commonly used functions to describe demand do not have a finite upper bound, one would normally assume that the budget constraint would be active.

Assuming that the budget constraint is active, the normal method of solving the problem is through the use of the Lagrange multiplier. The general Lagragian equation is

$$L = S + \Pi(K - B) \quad .$$

The solution comes from meeting the following conditions:

$$\frac{\partial L}{\partial \Pi} = 0 , \quad \frac{\partial L}{\partial R} = 0 , \quad \frac{\partial L}{\partial T} = 0 , \quad \frac{\partial L}{\partial \eta} = 0 \quad .$$

First, we will determine the optimal value of η by solving the $\frac{\partial L}{\partial \eta} = 0$ equation for η.

$$\frac{\partial L}{\partial \eta} = \frac{\partial S}{\partial \eta} + \Pi \frac{\partial K}{\partial \eta} = 0$$

and thus

$$\frac{\partial K}{\partial \eta} = -\frac{1}{\Pi}\frac{\partial S}{\partial \eta} .$$

From equation 4, we note that S is not a function of η ; hence, $\frac{\partial S}{\partial \eta} = 0$. From equation 5, we see that $\frac{\partial K}{\partial \eta}$ is

$$\frac{\partial K}{\partial \eta} = -\frac{A_R}{\eta^2} + \frac{\bar{y}(h_1 + h_2)}{2} .$$

Solving the $\frac{\partial K}{\partial \eta} = 0$ for η yields

$$\eta = \left[\frac{2 A_R}{\bar{y}(h_1 + h_2)}\right]^{1/2} . \qquad (6)$$

Next, we shall look at the condition $\frac{\partial L}{\partial \Pi} = 0$. From the general Lagragian equation the $\frac{\partial L}{\partial \Pi} = K - B$. This implies that $K = B$. Hence, solving $K = B$ for R as a function of T yields

$$R = \frac{1}{h_1}\left[B - \frac{(A_P + J)}{T} - \frac{A_R}{\eta} - \frac{\eta\bar{y}(h_1 + h_2)}{2}\right] + \frac{(\bar{x} - \bar{y})(T\bar{x} + 2\bar{x}\tau_P)}{2\bar{x}} . \qquad (7)$$

To determine the optimal values of R and T one could solve the $\frac{\partial L}{\partial T} = 0$ and the $\frac{\partial L}{\partial R} = 0$ simultaneously with equation 7. However, it is quite easy to solve by selecting several values of T, computing the associated values of R from equation 7, and then determining the values of S from equation 4 using the values of R and T. Throughout these computations, the optimal value of η should be used. A plot of S versus T may then be made to indicate the value of T that minimizes S. With that value of T, equation 7 is then evaluated for R.

The interpretation of the Lagrange multiplier, Π, is that it represents the decrease in the expected number of units backordered per year for a unit increase in the budget. The value of Π may be obtained from the solution of the $\frac{\partial L}{\partial R} = 0$. This yields

$$\Pi = \frac{1}{h_1 T} \int_R^\infty h(z, \tau_P + T)\, dz \quad . \tag{8}$$

After obtaining the optimal values of R and T that yield the minimum expected number of units backordered, Π may be evaluated from equation 8.

4. EXAMPLE

The following example is used to demonstrate the nature of the solutions presented by the model, and to explore the trade-off between procurement order and review costs and holding costs.

Let demand and the quantity of items returned to RFI_R be normally distributed with means of 1,000 units per year and 900 units per year, and standard deviations of 30 units per year and 50 units per year, respectively. Procurement leadtime, τ_P, is .5 years. The relevant costs are: $A_P = \$750$, $A_R = \$100$, $J = \$250$, $h_1 = \$200$, and $h_2 = \$20$. The two random variables, X and Y, are assumed to be independent. Also, demand during T is independent of demand during τ_P; and the quantity of items returned to RFI_R during T is independent of the quantity of items returned to RFI_R during τ_P. Therefore, the random variable Z is normally distributed with mean $100(\tau_P + T)$ and standard deviation equal to $[(\tau_P + T)\,3400]^{1/2}$.

Let $B = \$20{,}000$. Following the procedure outlined in section 3, the first step is to determine η. Evaluating equation 6 yields $\eta = 3.18 \times 10^{-2}$ years, which is approximately 12 days. Next, we solve equation 7 for various values of R for selected values of T. If $T = .1$ years, $R = 73$ units; if $T = .5$ years, $R = 133$ units; and if $T = 1$ year, $R = 163$ units. Next, using these values of R and T, determine the associated values of S. For this example, the general solution of S is

$$S = \frac{1}{T}\left[\sigma\,\phi\left(\frac{R-\bar{z}}{\sigma}\right) + (\bar{z}-R)\,\Phi\left(\frac{R-\bar{z}}{\sigma}\right)\right] ,$$

where

$$\phi(r) = \frac{1}{\sqrt{2\pi}}\,e^{-\frac{r^2}{2}} ,$$

$$\Phi(r) = \int_r^{\infty} \phi(x)\,dx ,$$

and $\bar{z}$ is the mean of Z, and σ is the standard deviation of Z.

The associated values of S are as follows:

T	R	S
0.1	73	123
0.5	133	21
1.0	163	22

From plotting these values, we see that the minimum S occurs near $T = .8$ years. Evaluating R for $T = .7, .8,$ and $.9$ years and evaluating the associated values of S, the following results were obtained.

T	R	S
0.7	146	21
0.8	152	20
0.9	158	22

Thus, the minimum expected number of units backordered per year is 20 when the review cycle is .8 years, the procurement order up to level is

152 units, and the cycle time for the receipt of repaired items is 3.18×10^{-2} years. The Lagrange multiplier, Π, may now be evaluated from equation 8. This yields $\Pi = 2.31 \times 10^{-3}$ units per dollar.

Various budget levels were considered to investigate the changes in the expected number of units backordered per year, S. The results are tabulated below. The mean demand during $\tau_P + T$, denoted as $\bar{\bar{x}}$, and the expected number of units backordered per review period, denoted as S_T, are also shown.

B	T	R	$\bar{\bar{x}}$	S	S_T
$10,000	4.0	276	450	45	180
$15,000	0.9	133	140	35	32
$20,000	0.8	152	130	20	16

The cost to operate the repair side of the system per year, with no review and procurement order costs, is approximately $6,000. With this budget level, the expected number of units backordered per year is 100. When the budget is increased to $10,000, the review cycle is very long because it is profitable to put the majority of the additional money into holding procured items, instead of sustaining the high procurement order and review costs frequently. As the budget is increased to $15,000, the review cycle decreases to .9 years, indicating that the system can afford to review and order procurement more often in order to achieve an economic balance between ordering and reviewing and holding costs. An increase of the budget by one-third to $20,000 yields only a slight decrease of the review period to .8 years, thereby putting most of the increased budget into safety stock.

5. SUMMARY

We have discussed a reparable-item inventory system with random demand. A periodic review policy was assumed for procured items, while inductions of carcasses were assumed to take place at regularly spaced intervals. As the accumulation rate of NRFI carcasses was assumed random, the repair batch sizes were also random. The objective was to determine the optimal procurement review period T, the optimal procurement order up to level R, and the optimal repair time period η, in order to minimize the expected number of units backordered per unit time, subject to an annual operating budget constraint. The model developed is an approximate model. The solutions presented are sensitive to the assumptions that η is small compared to T, and that the expected unit years of items backordered are small. Obviously, the model is also sensitive to the restrictiveness of the budget constraint.

As discussed in the introduction, the purpose of minimizing S subject to a budget constraint was to avoid postulating a shortage cost. However, even though a shortage cost may not be stated by a decision-maker or an item manager, it is implied as soon as a specific operating budget is established. We noted in section 3 that Π, defined as the Lagrange multiplier, may be interpreted as the decrease in the expected number of units backordered per year for a unit increase of the budget. Therefore, $\frac{1}{\Pi}$ is the shortage cost, which would yield the same decision rules if the more common criteria of minimum cost per unit time were used.

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13. ABSTRACT

A reparable-item inventory system has two sources of items to meet demands: from the procurement of new items, and from the repair of damaged or failed items. Further, the system contains two distinct inventories, one containing procured and repaired ready-for-issue items and the other containing those failed items awaiting repair. This thesis develops an approximate model of this general system assuming that the demand rate and return rate of failed items are probabilistic. A periodic review policy is assumed for procured items, and inductions of batches of failed items into repair are assumed to take place at regularly scheduled intervals of time. The model is structured as a single coordinated inventory system instead of two separate systems, one of procured items and one of repaired items. The optimal review period and "order up to" level for procured items are determined, along with the optimal time between repair batch inductions.

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UNCLASSIFIED

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Reparable						
Inventory Model						
Ready-for-Issue						
Non-Ready-for-Issue						

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